

Gödel brane

John D. Barrow^{1*} and Christos G. Tsagas^{1,2 †}

¹DAMTP, Centre for Mathematical Sciences, University of Cambridge,
Cambridge CB3 0WA, UK

²Department of Mathematics and Applied Mathematics, University of Cape Town,
Rondebosch 7701, South Africa

Abstract

We consider the brane-world generalisation of the Gödel universe and analyse its dynamical interaction with the bulk. The exact homogeneity of the standard Gödel spacetime no longer holds, unless the bulk is also static. We show how the anisotropy of the Gödel-type brane is dictated by that of the bulk and find that the converse is also true. This determines the precise evolution of the nonlocal anisotropic stresses, without any phenomenological assumptions, and leads to a self-consistent closed set of equations for the evolution of the Gödel brane. We also examine the causality of the Gödel brane and show that the presence of the bulk cannot prevent the appearance of closed timelike curves.

PACS number(s): 04.50.+h, 98.80.Cq

1 Introduction

The Gödel universe [1, 2] has been a well-studied example of the importance of understanding the global structure of space-time. It sheds light on the impact of cosmological rotation, the constraints imposed by space-time homogeneity, and the unexpected compatibility between general relativity and the presence of closed timelike curves. Whilst the Gödel universe is not a realistic model of our visible universe it may possess ingredients which illuminate the structure of the actual universe, and its stability properties reveal the extent to which its unusual causal structure may be expected to appear as a component of more general solutions of Einstein's equations. In this paper we will investigate the structure of a Gödel universe in a braneworld model.

Developments in string theory have inspired the construction of braneworld models, in which standard-model fields are confined to a surface that constitutes our 3-brane universe, while gravity propagates in all the spatial dimensions. A simple 5-D class of such models allows for a non-compact extra dimension via a novel mechanism for localization of gravity around the brane at low energies. This mechanism is the warping of the metric by a negative 5-D cosmological constant [3]. These models have been generalized to admit cosmological branes [4], and they provide an interesting arena in which to impose cosmological tests on extra-dimensional generalizations of Einstein's theory [5, 6].

*e-mail address: j.d.barrow@damtp.cam.ac.uk

†e-mail address: c.tsagas@damtp.cam.ac.uk

There are some interesting features of this scenario. An anisotropic stress is imprinted on the brane by the 5-D Weyl tensor, and this non-local field cannot be predicted by brane-bound observers since it includes 5-D gravitational-wave modes. The bulk equations are needed to determine the brane dynamics completely [4]. The full 5-D problem also involves choosing boundary conditions in the bulk. On the other hand, starting from a brane-bound viewpoint, any choice of anisotropic stress that is consistent with the brane equations will correspond to a bulk geometry, which can be locally determined in principle by numerical integration (or approximately, close to the brane, by Taylor expanding the Lie-derivative bulk equations given in [4]). However, numerical integration is very complicated (see [7] for the black hole case). Even if it can be successfully performed, it will not give the global properties of the bulk. The bulk geometry that arises for a given anisotropic stress may have unphysical boundary conditions or singularities (e.g., the bulk corresponding to a Schwarzschild black hole has a string-like singularity and a singular Cauchy horizon [8]). We have few exact bulk solutions to guide us in a study of cosmological anisotropy. One known solution [9] is the Schwarzschild-AdS bulk that contains a (moving) Friedmann brane, which is the exactly isotropic and homogeneous limit of an anisotropic universe. Some examples of spatially homogeneous Bianchi-type braneworlds have recently been given by Campos et al [10].

In section 2 we derive the kinematical properties of a non-expanding, shear-free, homogeneous, rigidly rotating brane of Gödel type. From these results we proceed in section 3 to determine their consequences for the structure of the bulk. Unlike the general situation, it is found that the Gödel brane creates a closed system of equations for the non-local bulk Weyl stresses. Finally, in section 4 we discuss the causal structure of the Gödel brane and how closed timelike curves persist despite the bulk effects.

2 Gödel brane

We study the dynamics of the Gödel universe in the context of the brane-world scenario, by assuming the presence of a rigidly rotating, non-expanding, shear-free and acceleration-free space-time, containing a single perfect fluid on the brane. Following [11], these are the minimum conditions that covariantly characterise the standard general relativistic Gödel model; we will also employ them to define what one might call a Gödel brane. In technical terms this means that, in addition to the perfect fluid assumption, we will impose the kinematical constraints $\Theta = 0 = A_a = \sigma_{ab}$, $\omega_a \neq 0$ and $\nabla_a \omega_b = 0$ on the brane. The scalar Θ describes the average volume expansion, A_a is the 4-acceleration, σ is the shear rate and ω_a is the vorticity rate associated with a chosen local timelike 4-velocity field u_a . Also, ∇_a is the covariant derivative operator with respect to the brane metric g_{ab} . For more details on the adopted formalism and notation the reader is referred to [5]. Having defined the Gödel brane, we can use the fully nonlinear covariant relations of [5] to obtain the equations that dictate the behaviour of the Gödel universe in the context of brane-world scenarios. Note that we do not consider explicit expressions for the brane and the bulk metrics. The embedding of braneworlds more complex than the original Minkowski and the standard FRW branes has been the subject of active research, but only a few complete anisotropic solutions are available in the literature (see [10] and references therein). Constructing the metrics of both the Gödel brane and of the host 5-dimensional bulk is a question that goes beyond the scope of this paper.

2.1 Kinematics

For a perfect fluid, the vorticity propagation and constraint equations are trivially satisfied. Of the remaining non-trivial kinematical relations, the generalised Raychaudhuri equation reads

$$\frac{1}{2}\kappa^2(\rho + 3p) - 2\omega^2 - \Lambda = -\frac{1}{2}\kappa^2(2\rho + 3p)\frac{\rho}{\lambda} - \frac{6}{\kappa^2\lambda}\mathcal{U}, \quad (1)$$

where $\kappa^2 = 8\pi/M_P^2$, \mathcal{U} is the dark energy density, λ is the brane tension and the terms on the right-hand side provide the local quadratic and bulk corrections. The generalised shear propagation equation takes the form

$$E_{ab} + \omega_{\langle a}\omega_{b\rangle} = \frac{3}{\kappa^2\lambda}\mathcal{P}_{ab}, \quad (2)$$

where E_{ab} is the magnetic component of the local Weyl tensor and angled brackets indicate the symmetric and trace free-part of tensors projected orthogonally to u_a on the brane. Here the corrections are due to the effective non-local anisotropic stress \mathcal{P}_{ab} arising from the free gravitational field in the bulk. Similarly, the generalised shear constraint reduces to

$$\frac{6}{\kappa^2\lambda}\mathcal{Q}_a = 0 \Rightarrow \mathcal{Q}_a = 0. \quad (3)$$

This means that the effective non-local energy flux vanishes identically, which reduces the available degrees of freedom in the bulk. Finally, the generalised gravito-magnetic constraint and the kinematics of the Gödel space-time guarantee that the magnetic part of the Weyl tensor vanishes:

$$H_{ab} = 0, \quad (4)$$

exactly as in the standard Gödel universe [11].

2.2 Conservation laws

When the matter has a perfect fluid equation of state, with energy density ρ and pressure p , the local energy and momentum conservation laws reduce to the constraints

$$\dot{\rho} = 0, \quad (5)$$

and

$$D_a p = 0, \quad (6)$$

respectively. Note that $D_a = h_a^b \nabla_b$, with $h_{ab} = g_{ab} + u_a u_b$, is the covariant derivative operator orthogonal to u_a on the brane. If the fluid is also barotropic, with $p = p(\rho)$, the former of the above implies that $\dot{p} = 0$ and the latter that $D_a \rho = 0$. Therefore, as in the standard Gödel model, both ρ and p are covariantly constant.

For the same matter content, and given the result (3), the non-local conservation equations become

$$\dot{\mathcal{U}} = 0, \quad (7)$$

and

$$\frac{1}{3}D_a \mathcal{U} + D^b \mathcal{P}_{ab} = 0. \quad (8)$$

Therefore, the bulk dark energy density remains constant in time, while its gradients are given by the transverse part of the effective non-local anisotropic stresses.

2.3 Ricci and Weyl curvature

The orthogonally-projected Ricci tensor \mathcal{R}_{ab} and Ricci scalar \mathcal{R} , which are obtained from the generalised Gauss-Codacci equation, satisfy the relations

$$\mathcal{R}_{\langle ab \rangle} = \frac{6}{\kappa^2 \lambda} \mathcal{P}_{ab}, \quad (9)$$

and

$$\mathcal{R} = 2(\kappa^2 \rho - \omega^2 + \Lambda) + \frac{\kappa^2 \rho^2}{\lambda} + \frac{12}{\kappa^2 \lambda} \mathcal{U}. \quad (10)$$

respectively. The latter is the generalised Friedmann equation on the Gödel brane.

The generalised propagation and constraint equations for the electric and magnetic Weyl components, assuming that the local matter is a barotropic fluid, are

$$\dot{E}_{ab} + \omega^c \epsilon_{cd\langle a} E_{b\rangle}{}^d = -\frac{3}{\kappa^2 \lambda} \left(\dot{\mathcal{P}}_{ab} + \omega^c \epsilon_{cd\langle a} \mathcal{P}_{b\rangle}{}^d \right), \quad (11)$$

$$\text{curl} E_{ab} = \frac{3}{\kappa^2 \lambda} \text{curl} \mathcal{P}_{ab}, \quad (12)$$

$$\text{D}^b E_{ab} = \frac{1}{\kappa^2 \lambda} \left(2\text{D}_a \mathcal{U} - 3\text{D}^b \mathcal{P}_{ab} \right) = -\frac{9}{\kappa^2 \lambda} \text{D}^b \mathcal{P}_{ab}, \quad (13)$$

with the latter equality obtained by means of (8), and

$$\kappa^2(\rho + p)\omega_a + 3E_{ab}\omega^b = -\frac{\kappa^2 \rho}{\lambda}(\rho + p)\omega_a - \frac{1}{\kappa^2 \lambda} \left(8\mathcal{U}\omega_a - 3\mathcal{P}_{ab}\omega^b \right). \quad (14)$$

Note that $\text{curl} W_{ab} = \varepsilon_{cd\langle a} \text{D}^c W_{b\rangle}{}^d$ for every orthogonally projected symmetric tensor W_{ab} and ε_{abc} is the projected alternating tensor.

2.4 Further constraints

Given that ω_a is covariantly constant on the brane, the projected divergence of Eq. (2) means that

$$\text{D}^b E_{ab} = \frac{3}{\kappa^2 \lambda} \text{D}^b \mathcal{P}_{ab}. \quad (15)$$

The above, with result (13), then imply that $\text{D}^b \mathcal{P}_{ab} = 0$, which in turn leads to

$$\text{D}_a \mathcal{U} = 0, \quad (16)$$

using the non-local momentum conservation law (8). Combined with the bulk energy conservation law given by (7), this result guarantees that the non-local energy of the Gödel brane is covariantly constant and acts as a (positive or negative) ‘dark’ cosmological constant in, say, Eq. (1).

Contracting (2) with ω_a , substituting the result into (14) and then contracting again with ω_a we obtain

$$\kappa^2(\rho + p) - 2\omega^2 = -\frac{\kappa^2 \rho}{\lambda}(\rho + p) - \frac{8}{\kappa^2 \lambda} \mathcal{U} - \frac{6}{\kappa^2 \lambda} \mathcal{P}_{ab} n^a n^b, \quad (17)$$

where n_a is the unit vector along the direction of the local rotation axis (i.e. $\omega_a = \omega n_a$, with $n_a u^a = 0$ and $n_a n^a = 1$). Substituted into (1), this gives

$$\kappa^2(\rho - p) + 2\Lambda = \frac{\kappa^2 \rho p}{\lambda} - \frac{4}{\kappa^2 \lambda} \mathcal{U} - \frac{12}{\kappa^2 \lambda} \mathcal{P}_{ab} n^a n^b, \quad (18)$$

which shows that, in contrast to the standard Gödel space-time, the local cosmological constant can now be positive. For example, in the simple case where $\mathcal{U} = 0 = \mathcal{P}_{ab}$ and $p = \rho$ we find that $\Lambda = \kappa \rho^2 / 2\lambda > 0$. Equations (17) and (18) combine to give

$$\kappa^2 \rho - \omega^2 + \Lambda = -\frac{\kappa^2 \rho^2}{2\lambda} - \frac{6}{\kappa^2 \lambda} \mathcal{U} - \frac{9}{\kappa^2 \lambda} \mathcal{P}_{ab} n^a n^b, \quad (19)$$

thus generalising the standard general relativistic constraints to Gödel branes (see [11]). Note that, on using the above, the projected Ricci scalar (see Eq. (10)) reduces to

$$\mathcal{R} = -\frac{18}{\kappa^2 \lambda} \mathcal{P}_{ab} n^a n^b, \quad (20)$$

and ensures that \mathcal{P}_{ab} determines both $\mathcal{R}_{\langle ab \rangle}$ and \mathcal{R} .

Equations (1)-(20) determine the dynamics of the Gödel brane by incorporating the local and non-local higher dimensional corrections. General relativity is recovered in the low-energy limit $\lambda^{-1} \rightarrow 0$. As it stands, the system is not closed since there is no propagation equation for the bulk anisotropic stress tensor \mathcal{P}_{ab} . The latter is customarily determined phenomenologically, by imposing certain physically and geometrically plausible constraints or by creating an arbitrary propagation equation for \mathcal{P}_{ab} .

3 Interaction between the brane and the bulk

3.1 Implications of the bulk

The presence of the bulk in brane-world models means that some of the standard Gödel constraints do not hold any more. The electric Weyl tensor, in particular, no longer satisfies the simple constraints of the standard Gödel space-time [11]. This is clearly demonstrated by the non-local energy and stress terms in Eqs. (2) and (11)-(13). The latter equations ensure the dependence of the local tidal forces on bulk quantities, such as \mathcal{U} and \mathcal{P}_{ab} . In fact, the time independence of the vorticity and Eq. (2) imply the relation

$$\dot{E}_{ab} = \frac{3}{\kappa^2 \lambda} \dot{\mathcal{P}}_{ab} \quad (21)$$

between the time derivatives of E_{ab} and \mathcal{P}_{ab} . This underlines an interesting effect of the bulk: the stationary nature of the Gödel universe is not guaranteed unless the non-local stresses are time-independent as well. This is also clear from the dependence of the Ricci curvature on \mathcal{P}_{ab} (see Eqs. (9) and (20)). Thus, both the local tidal forces and the local curvature will vary in time if the bulk anisotropy does so. Moreover, the presence of the bulk can also affect the homogeneity of the Gödel model, as relation (12) verifies. Therefore, rigidly rotating, non-expanding branes with zero shear and acceleration admit many more possibilities than the limiting case of the general relativistic Gödel universe.

3.2 Implications on the bulk

The fact that the bulk can influence the standard symmetries of the Gödel universe, such as its stationary nature and spatial homogeneity, is not entirely unexpected since the bulk introduces additional degrees of freedom to the model. Analogous modifications have also been identified in the brane-world generalisation of the Einstein static universe for example [12]. What is rather surprising is that the remaining symmetry in the Gödel brane is enough to dictate the exact behaviour of the bulk quantities, and in particular of the non-local anisotropic stresses, in a self-consistent way. In particular, consider the relation

$$\omega^c \epsilon_{cd\langle a} E_{b\rangle}{}^d = \frac{3}{\kappa^2 \lambda} \omega^c \epsilon_{cd\langle a} \mathcal{P}_{b\rangle}{}^d, \quad (22)$$

obtained by substituting E_{ab} in the right-hand side of the above from expression (2). Putting (22) back into Eq. (11) and using (21) we arrive at the following simple evolution law for the non-local anisotropic stresses

$$\dot{\mathcal{P}}_{ab} = -\omega^c \epsilon_{cd\langle a} \mathcal{P}_{b\rangle}{}^d, \quad (23)$$

which depends on the rotation of the local Gödel model. Accordingly, \mathcal{P}_{ab} will remain zero if it is zero initially, and the same also holds for \mathcal{U} (see Eq. (7)). In that case the behaviour of the Gödel brane approaches that of the standard Gödel model. Note that by substituting the above result into Eq. (11), the evolution law of the electric Weyl component reduces to

$$\dot{E}_{ab} = -\omega^c \epsilon_{cd\langle a} E_{b\rangle}{}^d, \quad (24)$$

ensuring the consistency of the system (21)-(24).

The existence of the evolution law (23) means that the Gödel brane is described by a closed and consistent set of equations, which incorporates the effects of the bulk anisotropy, without the need of extra phenomenological assumptions. This is a consequence of the time independence of the local vorticity vector, which leads to Eq. (21) first and then to the propagation equation (23). Obviously, the latter no longer holds if we relax the symmetry by allowing $\dot{\omega}_a \neq 0$.

4 Causality in the Gödel brane

In the last section we showed how the presence of the bulk can affect basic features of the Gödel universe, such as its stationarity. In view of this, it is worth examining whether the non-local effects can affect another key property of the Gödel model, namely the presence of closed timelike curves. This can happen in general relativistic Gödel-type cosmologies, that is in rigidly rotating homogeneous spacetimes with extra matter sources due to higher-order terms in the Lagrangian and in the presence of some string theory corrections [13, 14].

The induced field equations on the brane comprise a modified set of the standard Einstein equations with new terms that carry the bulk effects onto the brane [4, 5]

$$R_{ab} - \frac{1}{2} R g_{ab} = \kappa^2 T_{ab} + \frac{6\kappa^2}{\lambda} S_{ab} - \mathcal{E}_{ab} - \Lambda g_{ab}, \quad (25)$$

where the tensors S_{ab} and \mathcal{E}_{ab} respectively describe the local and nonlocal higher dimensional corrections. In our case T_{ab} has the standard perfect fluid form, which means that

$$S_{ab} = \frac{1}{12} \rho^2 u_a u_b + \frac{1}{12} \rho (\rho + 2p) h_{ab}, \quad (26)$$

with $S = S_a{}^a = \rho(\rho + 3p)/6$. Also,

$$\mathcal{E}_{ab} = -\frac{6}{\kappa^2\lambda} [\mathcal{U}(u_a u_b + \frac{1}{3}h_{ab}) + \mathcal{P}_{ab}] , \quad (27)$$

with $\mathcal{E} = \mathcal{E}_a{}^a = 0$. For a homogeneous brane the bulk has no temporal or spatial dependence either (see Eqs. (12), (13) and (22)). We can therefore treat this brane-bulk configuration as a homogeneous Gödel-type spacetime where the bulk acts as an extra source to the energy-momentum tensor of the matter [13]. The latter is given by

$$\mathcal{T}_{ab} = T_{ab} + \frac{6}{\lambda} S_{ab} - \frac{1}{\kappa^2} \mathcal{E}_{ab} , \quad (28)$$

where $\mathcal{T} = \mathcal{T}_a{}^a = 3p(1 + \rho/\lambda) - \rho(1 - \rho/\lambda)$, while the associated field equations read

$$R_{ab} = \kappa^2 \mathcal{T}_{ab} - \frac{1}{2} \kappa^2 \mathcal{T} g_{ab} + \Lambda g_{ab} , \quad (29)$$

given that $R = 4\Lambda - \kappa^2 \mathcal{T}$.

Adopting the signature conventions of [13] and using cylindrical coordinates, the metric of the aforementioned Gödel-type spacetime has the general form

$$ds^2 = \left[dt + \frac{4\omega}{m^2} \sinh^2\left(\frac{mr}{2}\right) d\phi \right]^2 - \frac{\sinh^2(mr)}{m^2} d\phi^2 - dr^2 - dz^2 , \quad (30)$$

where the parameter m specifies the class of the solution associated with the Gödel-type spacetime and also determines the causality of the model [13]. In particular, there will be no closed timelike curves if $m^2 \geq 4\omega^2$. Otherwise causality is not preserved. Following [13], we introduce an orthonormal frame relative to which the field equations (29) read

$$R_{AB} = \mathcal{T}_{AB} - \frac{1}{2} \kappa^2 \mathcal{T} n_{AB} - \Lambda n_{AB} , \quad (31)$$

where n_{AB} is the Minkowski metric with signature -2.¹ In the same frame the energy-momentum tensor of the effective fluid which incorporates the bulk effects is

$$\mathcal{T}_{AB} = \left[\rho \left(1 + \frac{\rho}{2\lambda} \right) + \frac{6\mathcal{U}}{\kappa^4\lambda} \right] V_A V_B - \left[p + \frac{\rho(\rho + 2p)}{2\lambda} + \frac{2\mathcal{U}}{\kappa^4\lambda} \right] h_{AB} + \frac{6}{\kappa^4\lambda} \mathcal{P}_{AB} , \quad (32)$$

with $V^A = \delta_0^A$ and $h_{AB} = n_{AB} - V_A V_B$. Substituting the above into Eq. (31) and then employing a straightforward calculation we obtain

$$R_{00} = \frac{1}{2} \kappa^2 \rho \left(1 + \frac{2\rho}{\lambda} \right) + \frac{6\mathcal{U}}{\kappa^2\lambda} + \frac{3}{2} \kappa^2 p \left(1 + \frac{\rho}{\lambda} \right) - \Lambda , \quad (33)$$

and

$$R_{11} = \frac{1}{2} \kappa^2 (\rho - p) - \frac{\kappa^2 \rho p}{2\lambda} + \frac{2\mathcal{U}}{\kappa^2\lambda} + \frac{6}{\kappa^2\lambda} \mathcal{P}_{11} + \Lambda , \quad (34)$$

with exactly analogous expressions for R_{22} and R_{33} . In addition, $R_{02} = 0$ which ensures that ω is constant [13]. The latter agrees with our original assumption of a rigidly rotating space. Finally, from [13] we have

$$R_{00} = 2\omega^2 \quad \text{and} \quad R_{11} = R_{00} - m^2 , \quad (35)$$

¹The reader is referred to [13] for further details on the formalism and the notation used in this section.

where the parameter m determines the class of the Gödel-type solution associated with our model. Results (34) and (35) combine to give

$$m^2 = 2\omega^2 - \frac{1}{2}\kappa^2(\rho - p) + \frac{\kappa^2\rho p}{2\lambda} - \frac{2\mathcal{U}}{\kappa^2\lambda} - \frac{6}{\kappa^2\lambda}\mathcal{P}_{11} - \Lambda. \quad (36)$$

At first it might seem that for certain brane-bulk configurations the right-hand side should be able to reach the desired value of $4\omega^2$ in which case the causality of the associated Gödel-type brane would have been preserved. On using the constraint (18), however, the above reduces to

$$m^2 = 2\omega^2 + \frac{6}{\kappa^2\lambda} \left(\mathcal{P}_{ab}n^an^b - \mathcal{P}_{11} \right). \quad (37)$$

Then, since we can always align our frame so that one of its spatial axes is in the direction of the vorticity vector, we have $\mathcal{P}_{11} = \mathcal{P}_{ab}n^an^b$. This implies that $m^2 = 2\omega^2$ just like in the standard Gödel model. In other words, the presence of the bulk cannot affect the causality of the brane and closed timelike curves still exist.

5 Discussion

Gödel's rotating cosmos has been one of the most intriguing solutions of the Einstein field equations. Since its discovery in 1949, the Gödel model has attracted continual attention because it contains closed timelike curves. This completely unexpected discovery not only stimulated the rigorous investigation of the global structure of spacetimes, but led to a reappraisal of our thinking about issues such as Mach's Principle and the possibility of time travel within relativistic theories. It has also led to a series of Gödel-type solutions [13]-[15] produced by the introduction of extra fields or of quantum-gravity corrections. Interestingly, in some of these solutions the causal pathologies usually associated with the Gödel universe do not occur.

It is generally believed that general relativity breaks down at very high energies and it is likely to be the limit of a more general theory. Developments in string theory suggest that gravity may be higher dimensional, confined to a 4-dimensional part of space-time at low energies. A number of these theories confine the matter to a 3-brane, while the gravitational field is free to propagate in the extra dimensions. In a popular realisation of this scenario, gravity is localised on a single 3-brane embedded within a 5-dimensional "bulk" space. This idea has triggered much work on the properties of these braneworld models, on their differences from their general relativistic counterparts, and on their potential for experimental test. Here, we have investigated the nature of a braneworld that exhibits the structure of the Gödel universe. Interesting differences emerge compared to the usual Gödel universe of general relativity due to the influence of the bulk stresses. The exact homogeneity of the Gödel space-time, for example, is no longer preserved unless the bulk is also static. Also, in contrast to the general braneworld case, the evolution formulae of the Gödel brane form a closed set of equations. In particular, the Gödel constraints enforce a single propagation equation for the non-local anisotropic stresses, which generally are determined on purely phenomenological grounds. We have also considered the causality of the Gödel brane by treating it as an effective Gödel-type spacetime where the bulk effects are represented as extra source terms in the energy momentum tensor of the matter. In particular, using the techniques of [13] we were able to show that the presence of the bulk does not affect the causality of the model and that closed timelike curves still exist. Note that analogous studies of Gödel-type spacetimes with extra sources, in the form of a scalar field or of higher-order gravity corrections, showed that in certain cases the causality problems of the

general relativistic Gödel cosmos can be avoided. Our analysis, however, shows that this not the case when dealing with the Gödel-type brane. We attribute this negative result to the highly constraint bulk of the Gödel braneworld. It is the strict symmetries of the Gödel brane which specify the properties of the bulk, leaving no residual freedom that can be used to remove the causality pathologies of the model.

Acknowledgements

CT was supported by a Sida/NRF grant, PPARC and DAA. He also wishes to thank the Centre for Mathematical Sciences at DAMTP, where this work started, for their hospitality.

References

- [1] K. Gödel, *Rev. Mod. Phys.* **21**, 447 (1949).
- [2] S.W. Hawking and G.F.R. Ellis, *The Large Scale Structure of Space-time*, (Cambridge University Press, Cambridge, 1973); I. Ozsváth and E. Schücking, *Class. Quantum Grav.* **18**, 2243 (2001).
- [3] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999).
- [4] T. Shiromizu, K.I. Maeda and M. Sasaki, *Phys. Rev. D* **62**, 024012 (2000).
- [5] R. Maartens, *Phys. Rev. D* **62**, 084023 (2000); R. Maartens, in *Reference Frames and Gravitomagnetism*, edited by J. Pascual-Sanchez et al. (World Scientific, 2001), p. 93; R. Maartens, gr-qc/0312059.
- [6] R. Maartens, D. Wands, B.A. Bassett and I.P.C. Heard, *Phys. Rev. D* **62**, 041301 (2000); D. Langlois, R. Maartens, M. Sasaki, and D. Wands, *Phys. Rev. D* **63**, 084009 (2001); C. Van de Bruck, M. Dorca, R.H. Brandenberger and A. Lukas, *Phys. Rev. D* **62**, 123515 (2000); D. Langlois, R. Maartens and D. Wands, *Phys. Lett. B* **489**, 259 (2000); D. Langlois, *Phys. Rev. Lett.* **86**, 2212 (2001); H. Kodama, hep-th/0012132; C. Gordon and R. Maartens, *Phys. Rev. D* **63**, 044022 (2001); E.J. Copeland, A.R. Liddle, and J.E. Lidsey, *Phys. Rev. D* **64**, 023509 (2001); N. Deruelle, T. Dolezel, and J. Katz, *Phys. Rev. D* **63**, 083513 (2001); V. Sahni, M. Sami and T. Souradeep, *Phys. Rev. D* **65**, 023518 (2002); H.A. Bridgman, K.A. Malik and D. Wands, *Phys. Rev. D* **65**, 04350 (2002); K. Koyama and J. Soda, *Phys. Rev. D* **65**, 023514 (2002); D.S. Gorbunov, V.A. Rubakov and S.M. Sibiryakov, hep-th/0108017.
- [7] T. Shiromizu and M. Shibata, *Phys. Rev. D* **62**, 127502 (2000); A. Chamblin, H.S. Reall, H. Shinkai and T. Shiromizu, *Phys. Rev. D* **63**, 064015 (2001).
- [8] A. Chamblin, S.W. Hawking and H.S. Reall, *Phys. Rev. D* **61**, 065007 (2000).
- [9] P. Binetruy, C. Deffayet, U. Ellwanger and D. Langlois, *Phys. Lett. B* **477**, 285 (2000); S. Mukoyhama, T. Shiromizu and K.I. Maeda, *Phys. Rev. D* **61**, 024028 (2000); P. Bowcock P.C. Charmousis and R. Gregory, *Class. Quantum Grav.* **17**, 4745 (2000).
- [10] A. Campos, R. Maartens, D. Matravers and C.F. Sopuerta, *Phys. Rev. D* **68**, 103520 (2003).

- [11] G.F.R. Ellis, in *General Relativity and Cosmology*, edited by R.K. Sachs (Academic Press, New York, 1971), p. 104; G.F.R. Ellis, in *Cargèse Lectures in Physics*, edited by E. Schatzmann (Gordon and Breach, New York, 1973), Vol. VI, p. 1; J.D. Barrow and C.G. Tsagas, gr-qc/0308067.
- [12] L.Á Gergely and R. Maartens, *Class. Quantum Grav.* **19**, 213 (2002).
- [13] M.J. Rebouças and J. Tiomno, *Phys. Rev. D* **28**, 1251 (1983)
- [14] A.J. Accioly, *Nuovo Cimento B* **100**, 703 (1998); J.D. Barrow and M.P. Dąbrowski, *Phys. Rev. D* **58**, 103502 (1998); P. Kanti and C.E. Vayonakis, *Phys. Rev. D* **60**, 103519 (1999).
- [15] M. Ryan and L. Shepley, *Homogeneous Relativistic Cosmologies*, Princeton UP, NJ, (1975).